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REVIEW

The Energy Method, Stability and Nonlinear Convection. By B. STRAUGHAN. Springer, 1992. 242 pp. DM98.

Osborne Reynolds was the first to employ the time rate of change of the total energy of an arbitrary disturbance to establish sufficient conditions for the stability of the underlying flow. Later studies by Orr, then by Serrin and by Joseph, mark the significant steps in understanding the virtues and weaknesses of this approach. This classical energy theory provides a value for the bifurcation parameter (i.e. Reynolds number, Rayleigh number, etc.) below which any disturbance must decay monotonically to zero. However, there tends to be quite a variation, from problem to problem, in how conservative this bound actually is. For example, in some supercritical problems (i.e. Bénard convection and narrow-gap Taylor–Couette flow) energy theory and linear theory merge at the point of bifurcation and thereby rule out any possibility of subcritical transitions. In this case, energy theory provides a tight bound in the sense that if the parametric value below which energy stability is guaranteed is violated, instability indeed results. Unfortunately, this is the exception rather than the norm in energy theory performance. An important case in point is parallel shearing flows, in which energy theory provides an exceedingly low bound (by more than an order of magnitude) for the observed subcritical, local and complicated transitional processes. As a consequence, for several decades attention has turned from global integral formulations to the current mechanistic and numerical studies.

The book by Straughan is the visible tip of a new industry (other workers in the field include Galdi, Mulone, Padula and Rionero) seeking to improve classical energy theory results by considering generalizations of 'energy' in the Lyapunov spirit. Typically, they are able to increase the energy stability boundary, but only at the cost of settling for 'conditional' energy stability results. Conditional energy stability theory (first advanced by Joseph & Hung 1971, and again in Joseph 1976*a*, *b*), has the added stipulation that decay is guaranteed only for those disturbances having initial energies below some derived threshold value.

The principal example studied in this book, drawn from Galdi & Straughan (1985), is convection with rotation. Since the Coriolis term does not work, it does not appear in the usual energy integral, and therefore classical energy theory results are identical for both rotating and non-rotating convection. However, the observations of Rossby (1969) and linear theory suggest that the critical Rayleigh number is a strictly increasing function of the rotation rate (measured by the Taylor number). Straughan commences his study, admittedly in heuristic fashion, by advancing a rather complicated generalized 'energy' for this problem. Reviewing the surrounding literature and other problems addressed by Straughan, it seems that the primary motivation for the choice of 'energy' comes from the linear part of the differential operator. The plotted results appear remarkably successful in paralleling the observations and linear theory. What are not adequately discussed or plotted are the amplitude restrictions on the 'energy' stability boundary. Indeed, only the 'nonlinear' results corresponding to disturbances of zero (!) initial energy are displayed. Using equation (5.36) in Galdi & Straughan we have determined a bound on their energy stability boundary for disturbances having initial energy amplitudes

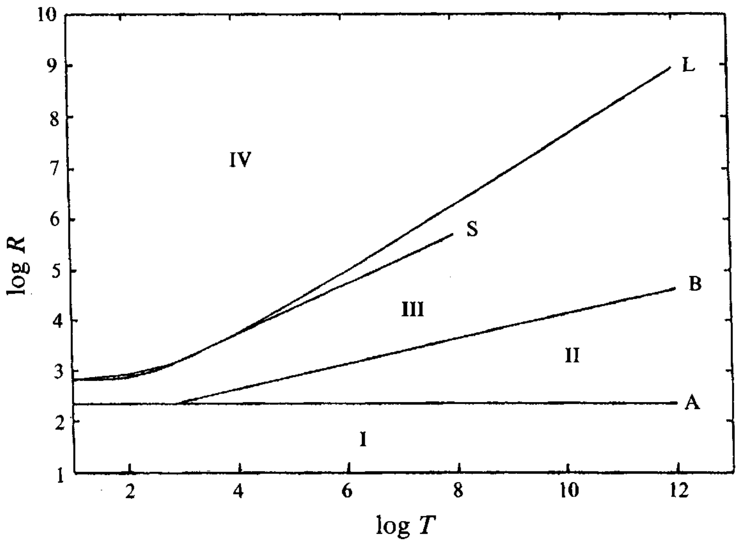

FIGURE 1. Rotating convection with $Pr > 1$. Taken from Straughan (1992), figure 6.2.

less than 10^{-6} times the scale amplitude (the viscous diffusion velocity). This result is plotted in figure 1, which is based on figure 6.2 of the book under review. Presented are four curves (A, B, S, L) and four regions (I, II, III, IV). Curve L is the linear stability curve, so that region IV is linearly unstable while regions I, II, III and any area between curves L and S are linearly stable. Curve A represents the classical energy theory results, and so region I is stable to disturbances of arbitrary size. Curve S represents Straughan's zero amplitude stability boundary, and curve B bounds from above region II wherein Straughan's 10^{-6} stability boundary must reside. Thus in region III the new theory unfortunately provides no information concerning the fate of disturbances having initial amplitudes in excess of 10^{-6}, the scale amplitude. One is led to question the value of curve S and bound B as useful 'nonlinear' results. Neither curve delimits the subcritical finite-amplitude observations of Rossby.

An order-one conditional energy bound would be of considerable interest, particularly in a parallel shear flow problem. Even more ambitious would be the determination of the role of the nonlinear terms (above certain amplitudes they can be stabilizing), which would reduce the task of finding amplitude bounds based on the presumption that they are entirely destabilizing. Here in rotating convection, for Straughan's 'energy' functional, the order-one bound is several orders of magnitude to the right of the graph above, and with a slope much less than the linear results, L. The purported goal is admirable, but it is the reviewers' opinion that a book on the topic should await the discovery of a formal path to an 'energy' functional from which results of clearer value are attained.

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